CHAPTER 9: AN OVERVIEW OF CAPITAL MARKET THEORY

Assigned problems are 1, 3, 6, 10, 12, 18, and 19. Read the chapter Appendix A.

I. Holding Period Returns

One share of stock is purchased today for $100. One year later the stock price is $116.91 and it has paid a dividend of $4.50.

Dollar profit = dividend + capital gain

Dollar profit = dividend + [ending price – beginning price] = dividend + $[P_1 - P_0]$

Dollar profit = 4.50 + [116.91 – 100] = 4.50 + 16.91 = $21.41

The total one-year Holding Period Return (HPR) is defined as:

$HPR = [\text{dividend} + P_1 - P_0]/P_0 = \text{dividend}/P_0 + (P_1 - P_0)/P_0$

HPR = 4.50/100 + (116.91 – 100)/100 = 0.045 + 0.1691 = 0.2141 or **21.41%**

This stock provided a total one-year Holding Period Return of 21.41%, consisting of a 4.5% *dividend yield* and a 16.91% *capital gains yield*.

II. Introduction to Risk and Return

Stock and bond returns are usually stated in terms of the mean (arithmetic mean) return and the standard deviation (the measure of risk or *dispersion* of individual returns around the mean return)

Shown below are the returns to U.S. T-bills and the Standard and Poors 500 Index (S&P 500) for 1981 through 1985. The mean return shown below is the mean arithmetic return (sum across all the years, then divide by number of years).

Year	T-bills	S&P 500
1981	0.1471	-0.0491
1982	0.1054	0.2141
1983	0.0880	0.2251
1984	0.0985	0.0627
1985	0.0772	0.3216
mean return	0.10324 or 10.324%	0.15488 or **15.488%**

The *variance* of any series of *n* investment returns *r* (in this course we only use *annual* returns), using realized or historical observations, is defined as:

$\sigma^2 = \frac{\sum_{i=1}^{n}(r_i - \bar{r})^2}{n-1}$, where we (1) calculate the mean annual return $\bar{r}$ for the series of *n* annual returns, (2) subtract this mean return from each annual return to calculate the *deviation* for the year, (3) take the square of this deviation, and (4) sum across all these squared deviations and then divide by n-1. The standard deviation is the square root of the variance:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(r_i - \bar{r})^2}{n-1}}$$

The following table reports the calculation of the annual deviations, squared deviations, and the sum of all squared deviations, i.e., $\sum_{i=1}^{n}(r_i - \bar{r})^2$.

Year	S&P 500	deviation	(deviation)^2
1981	-0.0491	-0.20398	0.041608
1982	0.2141	0.05922	0.003507
1983	0.2251	0.07022	0.004931
1984	0.0627	-0.09218	0.008497
1985	0.3216	0.16672	0.027796
Summation	0.7744	0	0.0863384

σ^2 = [sum of squared deviations]/(n-1) = 0.0863384/(5-1) = 0.0215846

$\sigma = [0.0215846]^{0.5}$ = 0.146917 or **14.69%** per year

For the 5-year 1981 through 1985 period, the mean annual return and standard deviation for the S&P Index was 15.49% and 14.69% per year, respectively.

Also, for this time period, the mean T-Bill return was 10.32% per year. T-Bills are usually defined as the *riskless* investment.

For 1981 through 1985, the mean annual *realized* risk premium for the S&P 500 Index was [15.49% – 10.32%] = **5.17%** per year.

III. Geometric Averages

A geometric average is *not* an arithmetic mean average, and the geometric average cannot be used to calculate standard deviations.

For the 5-year 1981 through 1985 period, the geometric average return for the S&P 500 Index is calculated as:

$r_{geo} = [(1+r_{81})(1+r_{82})(1+r_{83})(1+r_{84})(1+r_{85})]^{1/n} - 1$

$r_{geo} = [(1-0.0491)(1+0.2141)(1+0.2251)(1+0.0627)(1+0.3216)]^{1/5} - 1$

$r_{geo} = [1.9864]^{1/5} - 1 = 0.14713$ or **14.713%** per year

$1 invested at the beginning of 1981 grew to $1.9864 at the end of 1985.

IV. Historical Record of Investment Classes and Returns

The following table reports the arithmetic mean returns and standard deviations for several investment categories for the 1926 through 2002 period in the U.S. The mean annual inflation is also reported in the table.

Investment	Mean return	Standard dev.
Treasury bills	3.8%	3.2%
Long-term T-bonds	5.8%	9.4%
Long-term Corp. bonds	6.2%	8.7%
Large firm stocks	12.2%	20.5%
Small firm stocks	16.9%	33.2%
Inflation	3.1%	4.4%

Note that over this long time period (77 calendar years), investments with higher standard deviations (higher risk) earned higher average returns.

For example, large firm stocks earned a mean return of 12.2% per year, with a standard deviation of 20.5% per year.

In comparison to T-bills, we can say that large firm stocks earned an average *realized* risk premium of 12.2% – 3.8% = **8.4%** per year during the 1926-2002 time period. This is never to be interpreted as the *current* risk premium being 8.4% per year. No one knows what the actual *current* expected risk premium is!

<u>The normal distribution and stock returns:</u>

Histograms of annual returns to both large and small firms closely resemble a normal distribution or bell curve.

Assuming that stock returns can be described by a normal distribution, we expect that:

- Approximately 68.26% of the annual returns are in the range of plus/minus *one* standard deviation of the mean average return, e.g., [$\bar{r} + \delta$ to $\bar{r} - \delta$].
- Approximately 95.44% of the annual returns are in the range of plus/minus *two* standard deviations of the mean average return.
- Approximately 99.74% of the annual returns are in the range of plus/minus *three* standard deviations of the mean average return.

In the 1926-2002 period, large firm returns are described by a mean annual return of 12.2% and a standard deviation of 20.5%. We expect that approximately 68% of these returns fall within the range of plus/minus *one* standard deviation of the mean average.

$\bar{r} + \delta$ = 12.2% + 20.5% = **32.7%**

$\bar{r} - \delta$ = 12.2% - 20.5% = **-8.3%**

Thus we expect to find that approximately 68% of the annual returns fall within the range from –8.3% to +32.7%.