CHAPTER 6: ALTERNATE INVESTMENT TOOLS

Assigned problems are 1, 7, 10, 15, 19, and 25. Skip section 6.4 of the text.

I. Payback Period

The payback period is defined as the number of periods required to recover the *original cost* of the project.

Two *mutually exclusive* projects, X and Y, are presented in the Table below. Each costs CF_0=$1000 today (t=0), and produces the following annual cash flows:[1]

Project	CF_0	CF_1	CF_2	CF_3
X	-1000	1000	500	0
Y	-1000	500	500	2000

Project X: Payback period equals **1 year**. Complete payback is achieved when the year 1 cash flow $CF_1=1000$ is generated.

Project Y: Payback period equals **2 years**. CF_1 and CF_2 are each $500 and therefore they sum up to the original $1000 cost.

The usual practice is to choose projects with the shorter payback period. In this case, Project X would be chosen. Payback is generally inferior to the Net Present Value method due to the following reasons:

(1) Payback ignores the cash flows that occur after the payback period. For example, the CF_3 of $2000 for Project Y was ignored.

(2) Payback ignores the cost of capital and the time value of money.

Calculate the NPV of Projects X and Y, assuming a cost of capital of r=12% per year for each project.

$NPV_X = CF_0 + CF_1/(1+r) + CF_2/(1+r)^2$
$NPV_X = -1000 + 1000/(1+0.12) + 500/(1+0.12)^2 =$ **$291.45**

$NPV_Y = CF_0 + CF_1/(1+r) + CF_2/(1+r)^2 + CF_3/(1+r)^3$
$NPV_Y = -1000 + 500/(1+0.12) + 500/(1+0.12)^2 + 2000/(1+0.12)^3 =$ **$1268.59**

Project Y creates more wealth or value for the firm. If the firm can only choose one project, and thus chooses Project X based on the Payback rule, then the project actually having the higher NPV is rejected.

[1] If two or more projects are *mutually exclusive*, then only one of the several projects can be selected.

II. Discounted Payback and Average Accounting Return

We have already covered the Payback rule and its deficiencies. Discounted Payback still ignores the cash flows occurring after the Payback period. Average Accounting Return ignores the cost of capital and the time value of money.

These two tools will not be covered any further. Be aware of their deficiencies.

Capital Budgeting or project decision rules must take into consideration: (1) all cash flows and (2) the cost of capital and time value of money. The NPV method takes both (1) and (2) into account.

III. Internal Rate of Return (IRR)

A project, based upon its risk, only has *one* correct cost of capital. That cost of capital is then used as the *discount rate* to calculate the NPV of the project.

However, if we let the discount rate *r* in the NPV equation vary, we will eventually find some discount rate or cost of capital *r* that makes the NPV equal to ZERO.

IRR is defined as the discount rate that makes the NPV equal to ZERO.

A five year investment type project exists with the following net cash flows. The initial cost of this project is $1000 today (t=0).

t=0	t=1	t=2	t=3	t=4	t=5
CF_0 = -1000	CF_1 = 300	CF_2 = 300	CF_3 = 300	CF_4 = 300	CF_5 = 300

Based upon the risk of this project, we will let the cost of capital be r=12% per year. Note that the cost of capital is determined by the *market* and not by the firm. Investors in the market will require a return of *at least* 12% on any project or investment of similar risk. We can also refer to this r=12% as the opportunity cost of capital for this project. Assume here that the market for this scarce capital is highly competitive.

First, calculate the NPV of this project.

$$NPV = CF_0 + \frac{CF_1}{(1+r)} + \frac{CF_2}{(1+r)^2} + \frac{CF_3}{(1+r)^3} + \frac{CF_4}{(1+r)^4} + \frac{CF_5}{(1+r)^5}$$

$$NPV = -1000 + \frac{300}{(1+0.12)} + \frac{300}{(1+0.12)^2} + \frac{300}{(1+0.12)^3} + \frac{300}{(1+0.12)^4} + \frac{300}{(1+0.12)^5}$$

NPV = -1000 + 267.8571 + 239.1582 + 213.5341 + 190.6554 + 170.2281 = **$81.4329**

The NPV or wealth created is thus $81.43, meaning that the $1000 invested today creates an asset that is worth $1081.43.

Based on this project's *true* risk the correct cost of capital is r=12%. However, let's calculate the NPV using different discount rates in the calculation. The following table displays the results.

Discount Rate "r"	NPV
10.0%	$137.24
12.0%	**$81.43**
14.0%	$29.92
15.2382%	$0
16.0%	-$17.71

Note that the NPV=$0 if we let r=15.2382% in the NPV calculation. The Internal Rate of Return or IRR of this project is 15.2382%. The IRR is defined as the discount rate that makes NPV equal to zero.

$$0 = CF_0 + \frac{CF_1}{(1+IRR)} + \frac{CF_2}{(1+IRR)^2} + \frac{CF_3}{(1+IRR)^3} + \frac{CF_4}{(1+IRR)^4} + \frac{CF_5}{(1+IRR)^5}$$

For this project, the IRR is the root of a fifth order polynomial. Using a financial calculator, the IRR is easily obtained.[2]

C0 = -1000
C1 = 300
C2 = 300
C3 = 300
C4 = 300
C5 = 300
Use the IRR function to obtain **IRR = 15.2382%**

How to interpret and use the IRR correctly:

Now that we know that the IRR of the project is IRR = 15.2382%, what can we do with it? First we must address some critical issues concerning the IRR:

[2] To find the Payback Period for this project, simply calculate how long it takes to recover the original $1000 project cost. The first three cash inflows are each $300, so the payback is longer than three years and $100 remains to be recovered after CF_3 is received. Next look at the fraction of CF_4=300 that is needed to finish the payback. The project thus has a Payback Period of 3 + 100/300 = **3.3333 years**.

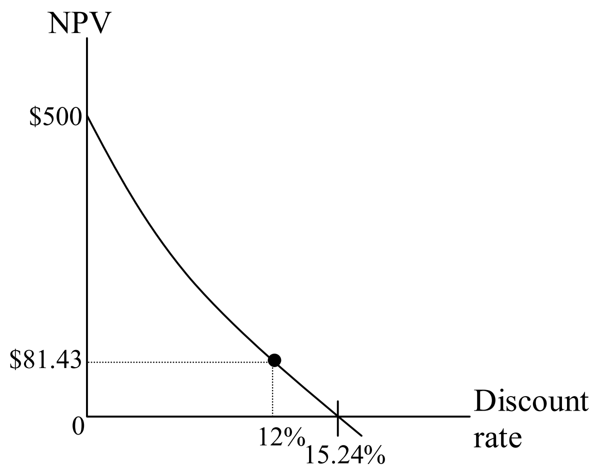
- While IRR is given in *percent*, the IRR is *not* the annual rate of return on the project. There is no correct annual rate of return on this project. IRR is just the discount rate that makes NPV=0.
- The IRR method assumes that the future project cash flows are *reinvested* at the same IRR rate, which may not be realistic. Only if reinvestment occurs at the IRR rate can we then state that the IRR is the annual rate of return. The NPV method assumes reinvestment at the cost of capital *r* rate, which is more realistic.
- A high IRR does *not* necessarily mean that the project should be accepted.

The correct cost of capital r (here, r=12% for the project under consideration) is commonly referred to as the *hurdle rate*. For "Stand-Alone" *investment type* projects:[3] [4]

(1) If the IRR is *greater* than the *hurdle rate* or cost of capital r, then *accept* the project since the NPV will be positive. Here the IRR is 15.24% and the cost of capital is r=12%, therefore the project would be accepted.

(2) If the IRR is *less* than the *hurdle rate* or cost of capital r, then *reject* the project since it would have a negative NPV.

The following graph illustrates the NPV *versus* discount rate relation:



Note that the IRR rule is rather redundant, after we have already calculated the NPV of the project. Also, NPV gives us a direct estimate of the value a project creates, while IRR does not. Why then, is IRR so popular among practitioners?

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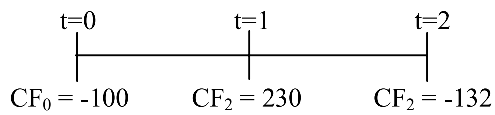
[3] "Stand Alone" means that the project is not competing with any other project. In other words, this project stands alone; the decision to accept or reject does not impact other projects than may be under consideration.

[4] "Investment type" projects are projects that require a net cash outflow at the beginning, followed by future cash inflows.

Individuals apparently have a preference and intuitive feel for items that are quoted as a *percentage* return. Surveyed managers often comment that NPV lacks an intuitive meaning, while the percentage return associated with IRR has meaning.

Multiple IRR Problem:

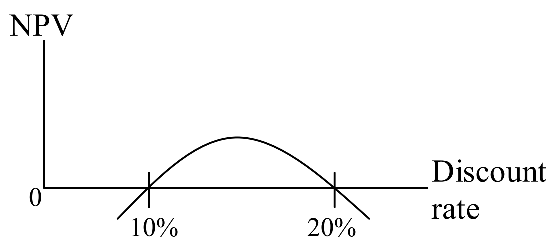
Projects having more than one change in the sign of cash flows may actually have several IRRs. Note the time line of the following project.



Finding the IRR here involves finding the roots of a quadratic equation:

$$0 = CF_0 + \frac{CF_1}{(1+IRR)} + \frac{CF_2}{(1+IRR)^2} \quad \rightarrow \quad 0 = -100 + \frac{230}{(1+IRR)} - \frac{132}{(1+IRR)^2}$$

There are two IRRs here: 10% and 20%. Most financial calculators will give an error message. A graph of this project's NPV *versus* discount rate is shown below:



When faced with multiple IRRs, use the NPV rule to make a decision. In this example, if the actual cost of capital or discount rate is between 10% and 20%, then the project has a positive NPV.

Scale Problem and *Incremental* IRR:

We are assuming there are no capital constraints. We will address the topic of constrained capital budgets in the Profitability Index section.

We have also treated projects as *stand alone*, meaning that the project is not competing with others. For *stand alone* projects, the IRR method can identify if the project has positive NPV. However, IRR is poor tool to use when you must compare several projects, e.g., choosing the best from some set of potential projects. If two projects are *mutually exclusive*, then you can choose only one of the two and not both.

Assume that a firm has the following two prospective projects, A and B. The projects are *mutually exclusive*. The cost of capital of each project is r=10%. Note that project B is simply a larger project that A.

Project	CF_0	CF_1	CF_2
A	-1000	750	750
B	-2000	1500	1300

NPV_A = -1000 + 750/(1+0.10) + 750/(1+0.10)^2 = **$301.65** , and **IRR = 31.87%**

NPV_B = -2000 + 1500/(1+0.10) + 1300/(1+0.10)^2 = **$438.01** , and **IRR = 26.41%**

Project B should be chosen since it has the higher NPV, e.g., the NPV of B is **$136.36** higher than A. Making a decision based solely on IRR would have resulted in the selection of project A (the lower NPV project).

While project A has the higher IRR, it creates less wealth than project B. The traditional IRR tool cannot compare two projects of different size or scale. We want to know if spending the additional $1000 for project B is worthwhile.

We can compute the *incremental* IRR. First, calculate the incremental cash flows by subtracting the project A (smaller project) cash flows from the project B (larger project) cash flows.

Project	CF_0	CF_1	CF_2
B	-2000	1500	1300
-(A)	-(-1000)	-(750)	-(750)
Incremental CF	-1000	750	550

Now, calculate the IRR (and NPV at r=10% for comparison purposes) of these *incremental* cash flows.

We obtain: **Incremental IRR = 20.6%** and **NPV = $136.36**. We already know from above that B has an NPV that is $136.36 higher than A.

The *additional* $1000 initial outlay (incremental CF_0) of project B over project A will generate an **IRR of 20.6%**. Since this incremental IRR is greater than the hurdle rate of r=10%, project B should be accepted and project A rejected.

IV. Profitability Index (PI)

The PI is defined as the ratio of the Present Value of future cash flows to the initial cost of the project. The formula is shown below:

$$PI \equiv \frac{\text{PV(cash flows)}}{\text{Initial cost}} = 1 + \frac{\text{NPV}}{\text{Initial cost}}$$

The following table illustrates the relation between PI and NPV:

PI	NPV
greater than 1.0	positive
equal to 1.0	zero
less than 1.0	negative

The PI measures the NPV generated per dollar spent on the project ("bang for the buck"). PI is useful when a firm's capital budget is *constrained* or limited.

- With a limited capital budget, the goal is to generate the maximum possible NPV from the limited amount of cash available to spend.
- Projects can be ranked from highest to lowest PI. Projects would then be chosen, beginning with the highest PI, until the capital budget is exhausted.

The following three projects A, B, and C exist. The cost of capital is r=12%.

Project	CF_0	CF_1	CF_2	CF_3
A	-10000	5000	5000	5000
B	-20000	7000	9000	11000
C	-25000	10000	12000	12000

$NPV_A = -10000 + 5000/(1+0.12) + 5000/(1+0.12)^2 + 5000/(1+0.12)^3 = \mathbf{-10000} + \mathbf{12009} = \mathbf{\$2009}$

$NPV_B = -20000 + 7000/(1+0.12) + 9000/(1+0.12)^2 + 11000/(1+0.12)^3 = -20000 + 21254 = \mathbf{\$1254}$

$NPV_C = -25000 + 10000/(1+0.12) + 12000/(1+0.12)^2 + 12000/(1+0.12)^3 = -25000 + 27036 = \mathbf{\$2036}$

For project A: the PV of future cash flows is $12009, the initial cost is $10000, and the NPV is $2009. Using the *two* PI formulas at the top of this page we can calculate the project PIs.

$PI_A = 12009/10000 = 1 + (2009/10000) = \mathbf{1.2009}$

$PI_B = 21254/20000 = 1 + (1254/20000) = \mathbf{1.0627}$

$PI_C = 27036/25000 = 1 + (2036/25000) = \mathbf{1.0814}$

Project A creates $0.2009 of NPV per $1 spent ("most bang for the buck").