

CHAPTER 5: HOW TO VALUE BONDS AND STOCKS

(Assigned problems are 1, 4, 5, 7, 8, 9, 13, 16, 17, 18, 21, 22, 23, 25, 26, 29, 31, and 33. Omit the Appendix to this chapter). This notes package contains two Addendums.

I. BOND VALUATION

Bonds are “Fixed Income” securities, since the cash flows that the bondholder will receive have been fixed or prespecified in the bond contract.

The current *fundamental* or *intrinsic* value of a bond (or any other financial asset) is equal to the Present Value or PV_0 of all future *expected* cash flows.¹

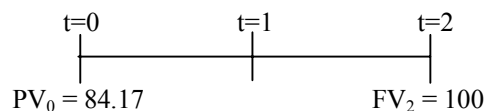
An investor’s actual return on a bond, for any holding period, comes in two forms:

- (1) the coupon yield (from the payment of coupon interest)
- (2) the capital gain (bond’s change in price)

Zero Coupon Bonds: a *zero coupon* bond will pay its stated face or par value at maturity. It pays no other future cash flows during its life. Zeroes are also known as *pure discount* bonds. The return here comes entirely as a *capital gain*.

Example: A zero coupon bond will mature exactly 2 years from today. It was sold or issued by the U.S. Treasury and therefore is free of default risk. The par or face value is \$100. This bond currently sells for \$84.17 in the market. What annual rate of return does the market expect on this bond?

From Chapter 4, we know: $PV_0 = FV_n/[1 + r]^n$. Here, $PV_0 = \$84.17$, $FV_2 = \$100$, and $n=2$ years. We need to solve for r . A time line of this bond is shown below:



$$PV_0 = FV_n/[1 + r]^n \rightarrow r = [FV_n/PV_0]^{1/n} - 1 = [100/84.17]^{0.5} - 1 = 0.09 \text{ or } \underline{\mathbf{9.0\%}}$$

Actually, anytime we calculate the annual yield r by using the current bond price and future payments, the r is referred to as the Yield-to-Maturity or YTM.

What will happen to this bond’s price tomorrow if the current market rate of interest or YTM on this and similar two-year bonds *falls* from 9.0% to 8.8%?

¹ *Intrinsic* value refers to a *private* estimate of value – a private estimate of Present Value. This is not the same concept as *market* value, which refers to the current price at which a bond or stock is trading for.

$PV_0 = FV_n/[1 + r]^n = 100/[1+0.088]^2 = \underline{\$84.48}$. Here, the bond is now priced to yield the new lower market rate of interest of 8.8% per year. As yields *fall*, bond prices *rise*. Bond yields (interest rates) and prices are always *inversely* related.

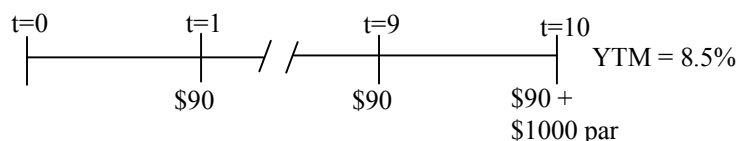
Coupon Bonds: most bonds pay coupon interest over their lives. Most coupon interest payments are paid every six months or semiannually. However we will cover *annual* coupon paying bonds first. We begin with an example.

Example: A bond was issued 10 years ago as a 20-year bond. Today, it has 10 years remaining until maturity. The face or par value of the bond is \$1000. The bond will pay an annual coupon interest payment equal to 9% of the par value.

Each year this bond will pay $(0.09)(1000) = \$90$ as a *coupon* interest payment. Over its remaining 10-year life, the bond will thus pay ten \$90 coupons and one amount of the \$1000 par at maturity.

Note: the coupon rate of 9% only determines the amount of the annual coupon payment. The coupon rate is completely *independent* of the actual YTM or current market rate of return on this bond. This bond will pay the \$90 coupons, *regardless* of what happens to the YTM. After all, this is a fixed income security.

At this time, market conditions are such that investors expect to earn a YTM of 8.5% on this and similar bonds. A time line of the bond is shown.



The ten \$90 coupons represent an *ordinary annuity* (as covered in Chapter 4) of ten \$90 cash flows. Then there is one *lump sum* of \$1000 at maturity. We calculate the Present Value or PV_0 of all these payments (the annuity and lump sum).

$$PV_0 = C \left[\frac{1}{r} - \frac{1}{r(1+r)^n} \right] + \frac{PAR}{(1+r)^n}$$

$$PV_0 = 90 \left[\frac{1}{0.085} - \frac{1}{0.085(1+0.085)^{10}} \right] + \frac{1000}{(1+0.085)^{10}}$$

$$PV_0 = (90)(6.561348) + 1000/(1+0.085)^{10}$$

$$PV_0 = 590.52 + 442.29 = \underline{\underline{\$1032.81}}$$

The coupon and par payments are worth \$590.52 and \$442.29 today ($t=0$), respectively. Thus investors are willing to pay \$1032.81 for this bond investment today and expect to receive a series of *eleven* future payments; ten annual coupon payments of \$90 each and one \$1000 amount ten years from today.

To work this problem on a financial calculator, these are the steps:

FV = \$1000 (the par value)

I = 8.5% (the current YTM)

N = 10 (number of coupons and number of periods until the par is received)

PMT = \$90 (the amount of the annual coupon)

P/Y = 1 (since periods and payments are made annually in this case)

The answer is **PV** -\$1032.81, and thus the price is \$1032.81. When the market price is greater than the par value, we say that the bond is selling at a *premium* (to par value).²

Bond Prices and Interest Rate Changes:

The previous bond is worth \$1032.81 today if the market requires a YTM=8.5%. Market rates of interest change continuously as economic conditions change.

When interest rates or yields *fall* (rise), bond prices *rise* (fall). In all cases, bond yields and prices are inversely related. The only way to obtain a *lower* (*higher*) yield or return on any fixed income investment is to pay *more* (*less*) for it.

In the last example, assume that the current market 8.5% YTM: (1) falls to 8%; (2) rises to 9%; and (3) rises to 9.5%.

(1) YTM falls from 8.5% to 8.0%

FV = \$1000

I = 8.0% (the new YTM)

N = 10

PMT = \$90

P/Y = 1

The answer is **PV** -\$1067.10, and thus the new price is **\$1067.10**. The price rises as the market YTM falls.

(2) YTM rises from 8.5% to 9.0%

² Note that in exactly one year, the bond has nine remaining years. If the market YTM is still 8.5%, then the price next year will be \$1030.60. Thus during course of one year, the bond pays \$90 in coupons and also *falls* in price by \$2.21. An investor's holding period return for the year is thus $[1030.60 - 1032.81 + 90] / 1032.81 = 87.79 / 1032.81 = 0.085$ or 8.5%, an amount that is equal to the YTM.

This bond's current value is found as follows (n=11.5 years and m=2):

$$PV_0 = C \left[\frac{1}{\text{YTM}/m} - \frac{1}{\text{YTM}/m (1 + \text{YTM}/m)^{n \cdot m}} \right] + \frac{\text{PAR}}{(1 + \text{YTM}/m)^{n \cdot m}}$$

$$PV_0 = 70 \left[\frac{1}{0.04} - \frac{1}{0.04(1 + 0.04)^{23}} \right] + \frac{1000}{(1 + 0.04)^{23}}$$

$$PV_0 = (70)(14.856842) + 1000/(1+0.04)^{23}$$

$$PV_0 = 1039.98 + 405.73 = \underline{\underline{\$1445.71}}$$

Calculator method (one of two ways)³:

FV = \$1000 (the par value)

I = 8% (the annual YTM)

N = 23 (number of 6-month or semiannual coupon periods until the par is received and the number of actual \$70 coupons)

PMT = \$70 (the amount of the semiannual coupon)

P/Y = 2 (since periods and payments are semiannual in this case; the calculator divides the YTM by m=2 and uses 4% as the effective 6-month rate to work this problem)

The answer is **PV** = -\$1445.71, and thus the price is **\$1445.71**

Note: working this problem as having annual coupons of \$140 each will always produce the wrong answer!

³ The alternate calculator method is to let P/Y=1 and I=4%. Everything else stays the same.

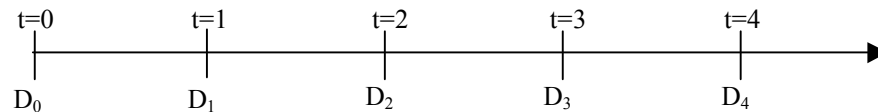
II. STOCK VALUATION

Current stock prices reflect today's *expectations* of the future cash flow performance of a corporation, as well as its expected risk. *Expectations* concerning the future can never be proven in the present.

We will assume that the firm accepts all new projects that increase the value of the firm, i.e., projects having a positive NPV. The excess cash that remains that can be paid out to the shareholders is referred to as **Free Cash Flow to Equity** or **FCFE**.

Here, we will assume that the FCFE is all paid out as a cash dividend. In reality, today firms pay out the FCFE by (1) dividends and (2) stock repurchases.

An analyst must forecast the firm's ability to pay out cash to stockholders in the future. The *fundamental* or *intrinsic* value of a stock is defined as the Present Value of all future *expected* FCFE (again, assume it is paid as dividends) that will be paid out. Observe the following time line of *expected* dividends.



We usually assume that the D_0 (if it exists) has just been paid out and is no longer part of the firm. The fundamental stock price P_0 can be modeled as follows⁴:

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t}, \text{ which is also expressed as } P_0 = \frac{D_1}{(1+r)^1} + \frac{D_2}{(1+r)^2} + \frac{D_3}{(1+r)^3} + \dots + \frac{D_t}{(1+r)^t} + \dots$$

The term r represents the market's required rate of return on the stock. In Chapter 10, we will learn how the r is determined.

Mature or Constant Growth Stocks:

This concept was first introduced in Chapter 4. With regard to the common stock of a mature firm, the dividend stream is expected to grow at a constant rate g as time passes. Everything associated with the firm is also expected to grow at the same rate g , including earnings, sales, cash flows, and the *stock price*.

Assume that a mature firm has just paid out dividend $D_0 = \$5$ per share to its common stock. This dividend stream is expected to grow at $g = 5\%$ per year. Dividends D_1 and D_2 paid exactly one and two years from today are expected to be the following:

⁴ A better term would be V_0 for *value*, rather than P_0 which is interpreted as *price*.

$$D_1 = D_0(1+g) = 5(1+0.05) = \$5.25$$

$$D_2 = D_0(1+g)^2 = 5(1+0.05)^2 = \$5.5125$$

The constant growth model is: $P_t = D_{t+1}/(r-g)$. The stock's value today, just after the D_0 has been paid is estimated to be: $P_0 = D_1/(r-g)$.

The required annual rate of return on this stock is $r=14\%$. The constant growth model introduced in Chapter 4 can be used to estimate the value of this stock.

$$P_0 = D_1/(r-g) = 5.25/(0.14-0.05) = 5.25/0.09 = \underline{\underline{\$58.33}}$$

Thus the Present Value of all expected future cash flows is estimated to be worth \$58.33 today. Does this mean that the stock actually sells for \$58.33 in the market today? If the analyst is correct about the fundamental value being \$58.33, then if the stock currently sells for \$50 it is *undervalued* in the market.

Also, according to this model, the stock's value next year, just after the $D_1=\$5.25$ is paid out should be:

$P_1 = D_2/(r-g) = 5.5125/(0.14-0.05) = \underline{\underline{\$61.25}}$. However, note that the following approach also works:

$$P_1 = P_0(1+g) = 58.33(1+0.05) = \underline{\underline{\$61.25}}$$

Changing the r or g in the constant growth model:

What happens to the current value P_0 above if the required rate of return *decreases* to $r=12\%$?

$P_0 = D_1/(r-g) = 5.25/(0.12-0.05) = \underline{\underline{\$75.00}}$. Thus the stock price should increase from \$58.33 to \$75.00.

If the required return r increases (decreases), the stock price decreases (increases).
If the cash flow growth rate g increases (decreases), the stock price increases (decreases).

Extensions of the constant growth model:

Here, we rearrange the model to explore other aspects.

$$P_0 = D_1/(r-g) \rightarrow \text{rearrange to obtain } \rightarrow r = D_1/P_0 + g$$

Using data from the prior example; $P_0=\$58.33$, $D_1=\$5.25$, $g=5\%$, and $r=14\%$.

$$r = D_1/P_0 + g = (5.25/58.33) + 0.05 = 0.09 + 0.05 = 0.14 \text{ or } \underline{\mathbf{14.0\%}}$$

This analysis tells us that the $r=14\%$ required rate of return on the stock comes as:

- (1) a 9% *dividend yield*
- (2) a 5% *capital gains yield*, e.g., an increase in share price of $g=5\%$ from $P_0=\$58.33$ to $P_1=\$61.25$.

Stocks with multiple or nonconstant growth stages:

The constant growth model; $P_0=D_1/(r-g)$, applies where the cash flows *following* D_1 at $t=1$ are expected to grow at a constant annual rate g .

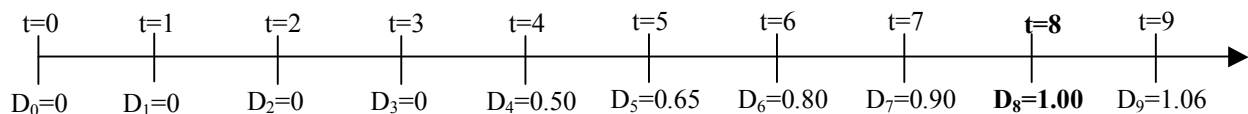
What about a firm that, like most firms, for the next 5 to 15 years, is not expected to grow at a constant rate? This same analogy can certainly be extended to firms that currently pay no dividends at all.

If the firm pays no dividends today (any cash flow is going toward reinvestment in the firm), then we must assume the firm will begin paying dividends at some time in the future and eventually mature, and thus grow at roughly the same rate as the overall economy.

Also, if the firm currently pays dividends, but these dividends are *not* expected to grow at constant rates during the near future, we still must assume that the firm does mature at some future point. A model such as the following is used, where at some time in the future, we will assume maturity and constant growth. Here, all dividends following the dividend at time $t+1$ grow at the annual rate g .

$$P_0 = \frac{D_1}{(1+r)^1} + \frac{D_2}{(1+r)^2} + \frac{D_3}{(1+r)^3} + \dots + \frac{D_t}{(1+r)^t} + \left[\frac{D_{t+1}}{r-g} \right] \left[\frac{1}{(1+r)^t} \right]$$

Example: Today, Cirrus Corp. is a growing firm that pays no dividends. It has earnings today, but it reinvests all of its earnings into new growth projects. You expect that Cirrus will pay dividends in the future, beginning with year 4 ($t=4$). The expected or forecasted dividends are $D_0=D_1=D_2=D_3=0$, $D_4=\$0.50$, $D_5=\$0.65$, $D_6=\$0.80$, $D_7=\$0.90$, and $D_8=\$1.00$. All dividends *after* year $t=8$ are expected to grow at a constant annual rate of $g=6\%$. The required rate of return on the stock is $r=10\%$ per year. The time line is shown below:



We next estimate today's intrinsic value of this stock. The valuation model can be set up as shown below. Note that the term, $D_8/(r-g)$, is very critical. This is the estimated stock price exactly 7 years from now. While D_9 is shown on the time line, it is not needed.

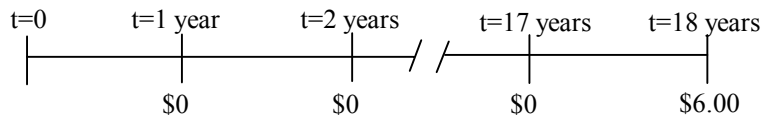
$$P_0 = \frac{D_4}{(1+r)^4} + \frac{D_5}{(1+r)^5} + \frac{D_6}{(1+r)^6} + \frac{D_7}{(1+r)^7} + \left[\frac{D_8}{r-g} \right] \left[\frac{1}{(1+r)^7} \right]$$

$$P_0 = \frac{0.50}{(1+0.10)^4} + \frac{0.65}{(1+0.10)^5} + \frac{0.80}{(1+0.10)^6} + \frac{0.90}{(1+0.10)^7} + \left[\frac{1.00}{0.10-0.06} \right] \left[\frac{1}{(1+0.10)^7} \right]$$

$$P_0 = 0.3415 + 0.4036 + 0.4516 + 0.4618 + [25][0.5132] = \underline{\underline{\$14.49}}$$

This private estimate of the intrinsic value of Cirrus stock is $P_0 = \$14.49$ per share. If the actual market price P_0 is below or above $\$14.49$ then you may want to buy or short the stock, respectively.⁵ The Cirrus stock price at $t=7$ years is expected to be $P_7 = D_8/(r-g) = \$25.00$, just after the dividend of $D_7 = \$0.90$ has been paid out. The $P_7 = \$25$ is deemed the *terminal price* (value when g becomes constant).

Example: XYZ Corp., a growth firm, is expected to pay its first dividend exactly 18 years from today. XYZ is forecasted to pay out nothing before that time. An analyst has estimated the following: $D_{18} = \$6.00$, $r = 14\%$, and $g = 7\%$. Calculate today's estimated intrinsic value.



Step 1: find the estimated stock price exactly 17 years from today.

$$P_{17} = D_{18}/(r-g) = 6/(0.14-0.07) = \underline{\underline{\$85.7143}}$$

Step 2: Find today's ($t=0$) intrinsic value. Discount the P_{17} price back to today to calculate P_0 .

$$P_0 = P_{17}/(1+r)^{17} = 85.7143/(1+0.14)^{17} = \underline{\underline{\$9.24}}$$

⁵ *Short selling* is selling a stock that you do not own, hoping that it will *fall* in price. Essentially, these shares are borrowed and identical shares must later be bought and returned to the owner. The goal is to buy back the shares at lower price than they were sold. In going short, you hope to sell high, and buy low. This is opposite the position of being *long* or owning the stock. In going *long* on a stock (having purchased a stock) you hope to buy low and sell high.

Estimating the permanent or long-run growth rate “g”:

We now discuss two common methods of estimating the permanent annual growth rate g .

1. $g = \text{expected annual inflation} + \text{expected annual real growth in GDP}$

For example, let expected inflation and GDP growth be 3% and 2.5%, respectively. The growth rate is then expected to be $g = 3\% + 2.5\% = 5.5\%$.

In the *long run*, it is unrealistic to assume that any firm can sustain a growth rate that exceeds the Gross Domestic Product growth of the economy. *Short run* growth rate estimates may certainly exceed the GDP growth rate.

2. $g = [\text{ROE} \times b]$

ROE is defined as the average future expected annual *Return on Equity* on the firm's capital expenditures. If a \$100 real investment generates \$16 per year forever for shareholders, then the investment has an annual $\text{ROE} = 16/100 = 16\%$. The firm's *retention* or *plowback* ratio b is defined as the proportion of economic earnings “E” that the firm retains and reinvests (*invest* here means *net* capital expenditures, or total capital expenditures minus depreciation). The firm thus has a *payout ratio* of $(1-b)$ to the stockholders.

- If $\text{ROE} = 16\%$ and $b = 0.40$, then $g = (0.16)(0.4) = 0.064$ or 6.4% per year.
- Using the constant growth model, you can replace “g” with $(\text{ROE})(b)$ and D_1 with $(1-b)(E_1)$ to obtain the following variant of the constant growth model:

$$P_0 = \frac{(1-b)E_1}{r - (\text{ROE})(b)}$$

Example: Cash Cow Inc., a mature firm, has $r=10\%$, next years earnings are expected to be $E_1=\$2$ per share, retention ratio $b=0.4$, and expected $\text{ROE}=16\%$ on its corporate investments. What is the intrinsic value of Cash Cow today?

$$P_0 = \frac{(1-0.4)(2.00)}{0.10 - (0.16)(0.4)} = \frac{1.20}{0.10 - 0.064} = \$33.33$$

Corporation Value and Growth Opportunities:

Any firm's current value can be decomposed into two portions:

Total firm value = PV of CFs from current operations
+ PV of future NPV growth activities

The first item above is often called the value of *assets in place*.

Example: An existing firm has no growth opportunities. The number of existing shares is 20 million. The firm invests just enough to keep the current assets in production (no *net* investment occurs, as total investment then equals the depreciation). In this case, the earnings equal the dividends paid and will be \$100 million per year forever (a perpetuity), so $g=0\%$. Required rate of return on the stock is $r=15\%$ per year.

First, estimate the value of the existing shares of stock:

$$P_0 = D_1/(r-g) = (100M/20M)/(0.15-0) = 5/0.15 = \underline{\$33.3333} \text{ per share}$$

Now a new project unexpectedly comes along. It costs \$15 million today and generates cash flows of \$5 million per year forever, beginning at $t=1$.

$$NPV_{\text{project}} = -15 + 5/0.15 = -15 + 33.3333 = \underline{\$18.3333 \text{ million.}}$$

Project's impact on existing stock price value: $18.3333M/20M = \underline{\$0.9166}$ per share

$$\text{New stock price: } P_0 = 33.3333 + 0.9166 = \underline{\$34.25} \text{ per share}$$

Price To Earnings or P/E Ratios:

We revisit the constant growth model. It is rearranged to solve for the P/E ratio.

$$P_0 = \frac{(1-b)E_1}{r - (ROE)(b)} \rightarrow \frac{P_0}{E_1} = \frac{(1-b)}{r - (ROE)(b)}$$

Note: $P_0/E_1 = (1-b)/(r-g)$. What about a very large mature firm that sells for a P/E of 60 to 80. Let $b=0.4$ and $r=10\%$. If the P/E is 70, then what is the market's consensus estimate of g ?

$P_0/E_1 = (1-b)/(r-g) \rightarrow 70 = (1-0.4)/(0.10-g) \rightarrow 0.10 - g = 0.6/70 \rightarrow g = 9.14\%$
This growth estimate is certainly unrealistic for a large firm, especially if the *nominal* GDP growth of the economy is 5 to 6% per year. What if the market begins to price the stock using a more realistic growth estimate of $g=6\%$?

$P_0/E_1 = (1-b)/(r-g) \rightarrow P_0/E_1 = (1 - 0.4)/(0.10 - 0.06) = 15$, which is close to the historical P/E average for mature stocks and the overall stock market (such as the S&P 500 index).

ADDENDUM 1 TO CHAPTER 5

This section presents a more integrated treatment of Corporate Finance and stock valuation.

First, we must define the following items:

- ROE The firm's expected average *Return On Equity* on its future real investments or capital expenditures.
- r The market's required rate of annual return on the common stock, based upon its level of risk; to be formally illustrated in Chapter 10.
- b Proportion of earnings (net income) that is reinvested into the firm as *Net Capital Expenditures*.⁶ *b* is also known as the *retention* ratio.
- g Permanent growth rate of a mature firm. Defined here as $g = (\text{ROE})(b)$.
- CE Capital Expenditures (the new positive NPV investments)
- INT Interest expense
- T Corporate income tax rate
- DEP Depreciation (never a cash flow, it is a non-cash expense)
- FCFE Free Cash Flow to Equity, the excess cash flow that can be paid out to the stockholders.

- EBITDA Earnings before Interest, Taxes, Depreciation, and Amortization; basically revenue minus operating costs.

Assume that we have a mature firm that is 100% financed by equity or common stock, and thus we allow the interest expense in this example is zero. The current figures apply for the fiscal year that has just ended.

Let CE = \$6000, DEP = \$5000, and EBITDA = \$9167. Also, let $r = 10\%$, ROE = 15%, $b = 40\%$, $T = 40\%$, and 10,000 shares of common stock exist.⁷

We want to estimate the *fundamental* or *intrinsic* value of this firm. We will just assume that the current FCFE_0 has just been paid out as a share repurchase and/or cash dividend. We further simplify this example by assuming that there is no investment in *Net Working Capital* (short term assets as shown in Chapter 7)

⁶ If total capital expenditures are equal to depreciation, then there are no *net* capital expenditures. Net capital expenditures are equal to total capital expenditures *minus* depreciation.

⁷ The $r = 10\%$ represents the firm's cost of equity capital.

$$FCFE_0 \equiv [EBITDA - DEP - INT][1 - T] + DEP - CE$$

$$FCFE_0 = [9167 - 5000 - 0][1 - 0.40] + 5000 - 6000$$

$$FCFE_0 = 2500 + 5000 - 6000 = \underline{\$1500} \text{ (the \$2500 amount is the Net Income)}$$

$$\text{Net Capital Expenditures} \equiv \text{Total CE} - \text{DEP} = 6000 - 5000 = \underline{\$1000}$$

Retention ratio = $b = [\text{Net CE}/\text{Net Income}] = 1000/2500 = \underline{0.40}$ or 40%, note that b was already given but this is where it originates.

Calculate this mature firm's permanent or long-run annual growth rate g :

$g = (\text{ROE})(b) = (0.15)(0.4) = \underline{0.06}$ or 6% per year. Note that new Capital Expenditures are expected to earn an average return of 15% per year, while $r = 10\%$, meaning that the capital expenditures are expected to generate a higher return than the cost of capital.

In fact, all of the above figures used in the calculation of $FCFE_0$ are forecasted to grow by $g = 6\%$ per year.

Next year's ($t = 1$) forecasted $FCFE_1$, based on what we assume today:

$$FCFE_1 = FCFE_0(1 + g) = 1500(1 + 0.06) = \underline{\$1590}$$

Now estimate the current ($t=0$) value of this firm's equity:

$$\text{Value} = P_0 = FCFE_1/(r - g) = 1590/(0.10 - 0.06) = \underline{\$39,750}$$

Fundamental value is thus $[\$39,750/10,000 \text{ shares}] = \underline{\$3.975}$ per share

If the current *market* price were actually less than (greater than) $\$3.975$, then we would believe that this stock is undervalued (overvalued).

Be aware that this is only a forecast or estimate of the current value. We don't know any of these numbers or parameters in the above models with certainty.

RWJ textbook: the text gives $P_0 = D_1/(r - g)$. This is technically accurate if the firm pays out all of its FCFE as a dividend; however, this scenario is not likely.

Calculate the P/E ratio, defined here as $P/E = P_0/NI_1$:

$P_0/E_1 = 39,750/[(2500)(1.06)] = \underline{15}$ for the *leading* P/E ratio. This firm is expected to trade at a price that is 15 times next year's earnings or Net Income.

Growth versus No Growth:

Assume that the firm has no growth opportunities. In this scenario, $CE = DEP$, for each year as the firm invests just enough to offset the depreciation, i.e., there are no Net Capital Expenditures and also $g = 0$ and $b = 0$.

$$FCFE_0 \equiv [EBITDA - DEP - INT][1 - T] + DEP - CE$$

$$FCFE = [9167 - 5000 - 0][1 - 0.40] + 5000 - 5000 = \underline{\$2500} \text{ for every year}$$

$$\text{Value as no growth} = FCFE/(r - g) = 2500/(0.10 - 0) = \underline{\$25,000}$$

The previous value of the firm, assuming 6% annual growth, was \$39,750. Now we can compare the growth *versus* no growth values of this firm. Any firm's value consists of the following:

$$\text{Total Value} = PV(\text{no growth}) + PV(\text{future NPV growth opportunities})$$

$$39,750 = 25,000 + NPVGO$$

$$\text{Thus the } \underline{NPVGO = \$14,750}$$

Note that if $r = ROE$, then future investments have zero NPV and will add no value over the no growth value. Growth opportunities only add value if they have a positive NPV, here meaning that $ROE > r$. Conversely, if $ROE < r$, then new investments will *destroy* value!

Also note one very important item from these Chapter 5 notes: equity valuation takes into consideration *all of today's expectations of future performance*.

ADDENDUM 2 TO CHAPTER 5

Corporate valuation using Free Cash Flows (FCFs):

The most commonly used valuation tool by financial analysts is the **Free Cash Flow (FCF)** valuation model, which is somewhat different from the FCFE model covered in Addendum 1, although there are many similarities. The FCFE model is used to *value only the firm's equity*, while the FCF model is used to *value the entire corporation* (the sum of debt and equity). Once the entire corporate value is estimated, then the debt value is subtracted out, and then what remains is thus the estimate of the equity value.

At this point in the course, there are some disadvantages with covering the Free Cash Flow or FCF model. First, the discount rate or cost of capital that must be used is the *weighted average cost of capital* or **WACC** — which is not fully covered until Chapter 12.⁸ Free Cash Flow (FCF) is roughly defined as the cash that the firm can pay out to its investors, both stock and debt owners, after the firm has met its obligations and investment needs.

The FCF can be expressed by the equation shown below — it is important to compare this FCF tool below to the Capital Budgeting cash flow analysis that we will cover in Chapter 7, as the FCF method values the entire firm, while Capital Budgeting methods in Chapter 7 value a single project.

$$\text{FCF}_i = [\text{EBIT}]_i [1 - \text{tax rate}] + \text{Depreciation} - \text{Capital Expenditures} \\ - \Delta \text{Net Working Capital} - \text{Principal Repayments} + \text{New Debt Issues}$$

The most common practice is to estimate ten individual annual FCFs and then assume maturity or constant annual growth following year 10 at the rate g . Thus the FCF valuation model most commonly used appears below.

$$V_0 = \frac{\text{FCF}_1}{(1 + \text{wacc})^1} + \frac{\text{FCF}_2}{(1 + \text{wacc})^2} + \frac{\text{FCF}_3}{(1 + \text{wacc})^3} + \dots + \frac{\text{FCF}_{10}}{(1 + \text{wacc})^{10}} + \left[\frac{\text{FCF}_{11}}{\text{wacc} - g} \right] \left[\frac{1}{(1 + \text{wacc})^{10}} \right]$$

The constant growth model as used above obtains $V_{10} = \text{FCF}_{11} / (\text{wacc} - g)$. V_0 is the total *intrinsic value* (enterprise value or equity *plus* debt) of the firm. Subtract the current debt value from V_0 to obtain the estimated equity value.

⁸ WACC represents a weighted average cost of capital for the firm, or what the firm's current or existing mix of both debt and equity financing currently cost the firm. In Chapter 12, we will cover this concept in detail.

For firms that need infusions of cash for the next few years, before the FCFs (hopefully) eventually become positive, the FCF model allows you to use *negative* FCFs.

Example: Today, Trillium Corp. is a growing firm that pays no FCF. It has earnings today; however, after reinvesting all of its earnings into new growth projects, it must still borrow additional cash to meet its investment needs. You expect that Trillium will eventually pay positive FCFs in the future, beginning with year 4 ($t=4$). The expected or forecasted FCFs (all given in millions of US\$) are $FCF_1=-\$0.80$, $FCF_2=-\$0.50$, $FCF_3=-\$0.20$, $FCF_4=\$0.50$, $FCF_5=\$0.65$, $FCF_6=\$0.80$, $FCF_7=\$0.90$, $FCF_8=\$1.00$, $FCF_9=\$1.10$, and $FCF_{10}=\$1.20$. All FCFs *after* year $t=10$ are expected to grow at a constant annual rate of $g=6\%$. The WACC of this firm is 10% per year. The firm has \$2 million of debt outstanding (at current market value).

We next estimate today's intrinsic value of this firm. The valuation model can be set up as shown below. Note that the *terminal value* term, $FCF_{11}/(r-g)$, is very critical, as this is the estimated firm value exactly 10 years from now.

$$V_0 = \frac{FCF_1}{(1+wacc)^1} + \frac{FCF_2}{(1+wacc)^2} + \frac{FCF_3}{(1+wacc)^3} + \dots + \frac{FCF_{10}}{(1+wacc)^{10}} + \left[\frac{FCF_{11}}{wacc-g} \right] \left[\frac{1}{(1+wacc)^{10}} \right]$$

$$V_0 = \frac{-0.80}{(1+0.10)} + \frac{-0.50}{(1+0.10)^2} + \frac{-0.20}{(1+0.10)^3} + \frac{0.50}{(1+0.10)^4} + \frac{0.65}{(1+0.10)^5} + \frac{0.80}{(1+0.10)^6}$$

$$+ \frac{0.90}{(1+0.10)^7} + \frac{1.00}{(1+0.10)^8} + \frac{1.10}{(1+0.10)^9} + \frac{1.20}{(1+0.10)^{10}} + \left[\frac{1.272}{0.10-0.06} \right] \left[\frac{1}{(1+0.10)^{10}} \right]$$

$$V_0 = -0.7273 - 0.4132 - 0.1503 + 0.3415 + 0.4036 + 0.4516 + 0.4618 + 0.4665$$

$$+ 0.4665 + 0.4627 + [31.80][0.3855] = \mathbf{\$14.0237 \text{ million}}$$

This private estimate of the *intrinsic* value of Trillium is $V_0 = \mathbf{\$14.0237 \text{ million}}$. Since this firm has \$2 million of debt outstanding (at current market value), the firm's equity is thus estimated to have an intrinsic value of $\$14.0237 - \$2 = \mathbf{\$12.0237 \text{ million}}$. If there are 100,000 shares of common stock, then the stock is estimated to be worth \$120.24 per share.