

CHAPTER 4, APPENDIX A: FINANCIAL MARKETS AND NPV

(Assigned problems are 1, 2, 3, and 5)

Our examples will use a 1-period model (one year, beginning now), while holding risk as constant, therefore representing an abstraction. Yet these examples explain why financial markets exist and how particular individuals choose to invest and consume their resources.

Financial or capital markets allow individuals to shift their consumption patterns across time.

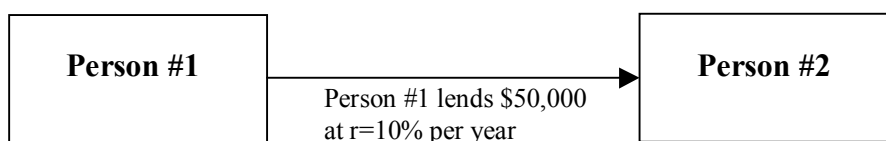
Individuals have a preference for current consumption with their money. In order to delay or postpone consumption, they expect to be compensated with a real rate of return. “Investing” is defined as postponing some consumption today, hoping to consume a greater real amount in the future.

Example No. 1: We are given the following two individuals. Assume that they can borrow and lend with one another at an interest rate of $r=10\%$ per year.

Person #1 has \$100,000 now and he receives no income next year. He wants to consume \$50,000 now. The remaining \$50,000 will be invested at $r=10\%$ and then consumed next year. Person #1 expects to be compensated for delaying consumption.

Person #2 has no cash today. However, he will receive exactly \$100,000 next year. Person #2 does want to consume \$50,000 now. He will have to borrow this \$50,000 amount at $r=10\%$ and repay the loan next year.

Assume that no financial intermediaries (banks) exist here, so individuals must borrow/lend from each other. Person #2 borrows the \$50,000 from Person #1 for one year at $r=10\%$.

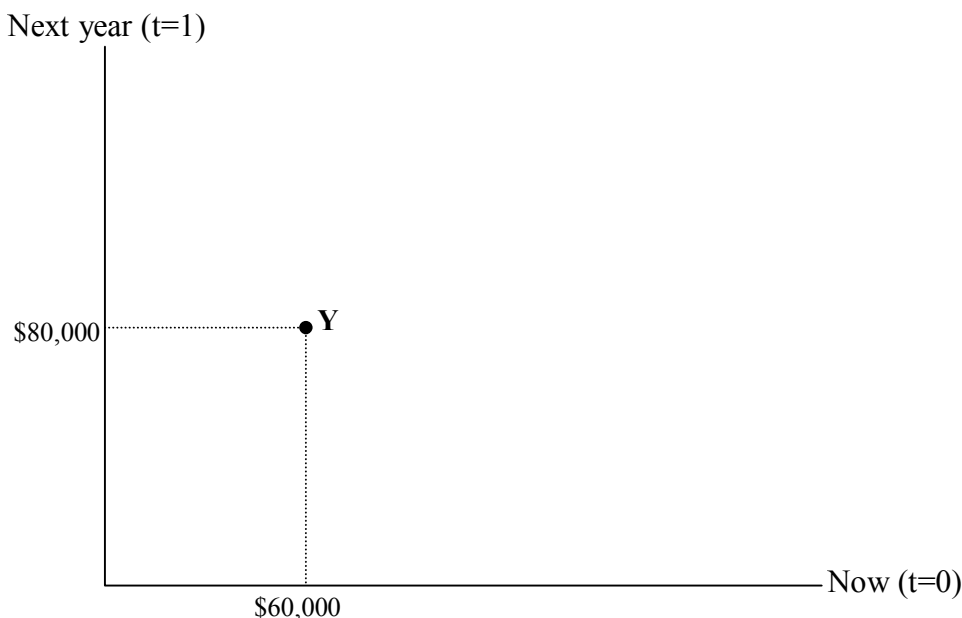


	Person #1	Person #2
Today (t=0)	Has \$100,000 now Consumes \$50,000 Lends remaining \$50,000	Borrows \$50,000 now Consumes all \$50,000
Next year (t=1)	Receives no income, only receives $(50,000)(1+0.1) =$ \$55,000 when loan is repaid. Has \$55,000 to consume	Receives the \$100,000 income Repays loan $(50,000)(1+0.1) =$ \$55,000 Has \$45,000 to consume

EXAMPLES OF INTERTEMPORAL CONSUMPTION

CHOICES:

Example No. 2: Financial or capital markets do not exist. Mary has \$60,000 today ($t=0$) and will receive \$80,000 in exactly one year ($t=1$). The graph below illustrates Mary's "endowment" point "Y" of having \$60,000 now and receiving \$80,000 next year.



Here, if no capital or financial market exists, then Mary must consume \$60,000 now and consume \$80,000 next year. Without a financial market, there is no borrowing or lending, and therefore consumption cannot be shifted across time. Mary's "intertemporal" consumption possibilities are fixed at the point "Y".

Example No. 3: Extending Example 2 from above, assume that a financial market exists and Mary can borrow and lend money at an annual interest rate of $r=10\%$.

Many intertemporal consumption choices now become available. Five possible choices are shown and discussed below.

- (1) **Consume** \$60,000 now ($t=0$) and **consume** \$80,000 next year ($t=1$) as in Example 2 above. If Mary desires this consumption pattern then there is no need to borrow or lend to shift consumption patterns across time. Mary just consumes at point "Y" as shown in Example No. 2 graph above.
- (2) **Consume** the **maximum** amount possible today, leaving absolutely no consumption for next year (ignore the unfeasibility of this choice, this is just an abstract example).

Today ($t=0$): **Consume** the existing \$60,000 today and also borrow against next year's \$80,000 income and **consume** all that is borrowed. Given $r=10\%$, how much can Mary borrow against next year's \$80,000 income? Each \$1 borrowed is \$1.10 repaid next year.

Mary can borrow $[80,000/(1+r)] = 80,000/(1+0.1) = \$72,727.27$ today. This $\$72,727.27$ amount is called the Present Value (PV) of next year's $\$80,000$ income.

Next year (t=1): Mary must pay back the $\$72,727.27$ loan. She must repay this loan with 10% interest, so she will pay $(72,727.27)(1+0.1) = \$80,000$ next year. This is equal to next year's income, so there is no remaining money with which to consume.

Mary's consumption now and next year is as follows. **Today:** Consume $\$60,000 + \$72,727.27 = \underline{\$132,727.72}$. **Next year:** Consume ZERO.

- (3) **Consume** nothing today and **consume** the **maximum** amount possible next year.

Today: **Consume** nothing today and thus invest or lend all of the current $\$60,000$ for one year at $r=10\%$. Note that this amount will grow to $(\$60,000)(1+0.1) = \$66,000$ next year ($t=1$).

Next year: ($t=1$) Mary receives her $\$80,000$ income, and she also receives $\$66,000$ as a result of having invested the $\$60,000$ at $t=0$.

Mary's consumption now and next year is as follows. **Today:** Consume ZERO. **Next year:** Consume $\$80,000 + \$66,000 = \underline{\$146,000}$.

- (4) **Consume** $\$90,000$ today. This requires borrowing $\$30,000$ against next year's income.

Today (t=0): **Consume** $\$90,000$ today, consisting of the existing $\$60,000$ and $\$30,000$ borrowed at $r=10\%$. Given that up to $\$72,727.27$ can be borrowed in (2) above, borrowing $\$30,000$ today will leave some consumption for next year.

Next year (t=1): Mary receives her $\$80,000$ income. She must repay the $\$30,000$ loan with 10% interest, so she will repay $(30,000)(1+0.1) = \$33,000$ next year. The amount of money that remains is $\$80,000 - \$33,000 = \$47,000$.

Mary's consumption now and next year is as follows. **Today:** Consume $\$90,000$. **Next year:** Consume $\$47,000$.

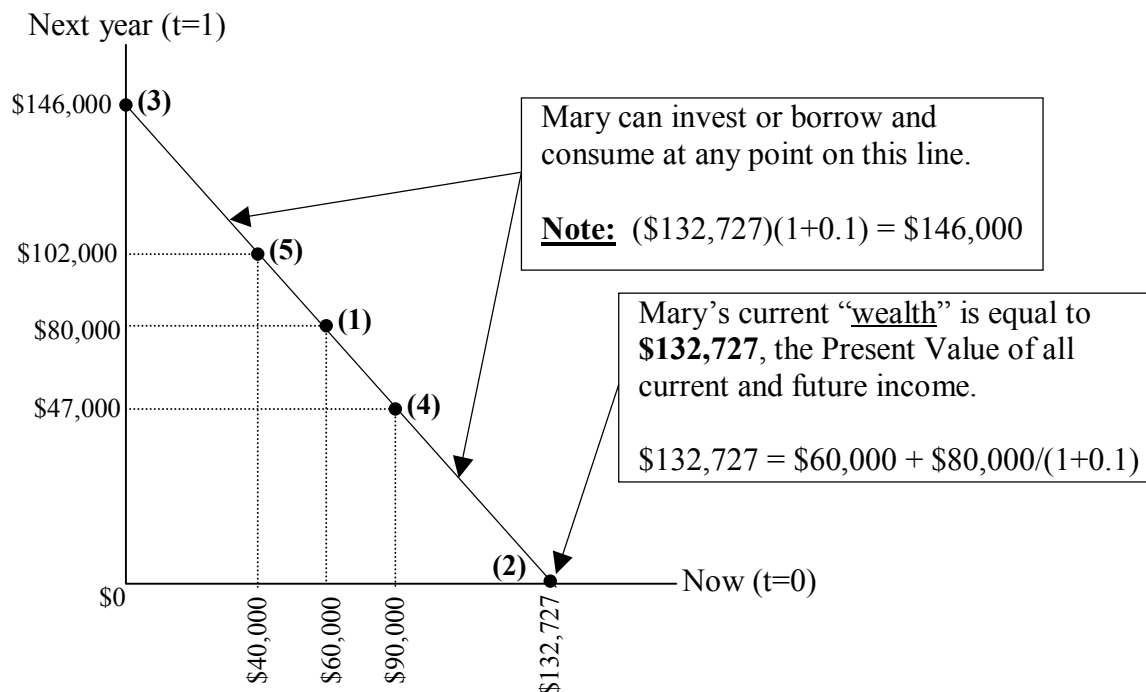
- (5) **Consume** $\$40,000$ today. Since $\$60,000$ exists, the remainder is invested.

Today (t=0): **Consume** $\$40,000$ today and invest the remaining $\$60,000 - \$40,000 = \$20,000$ at $r=10\%$.

Next year (t=1): Mary receives her $\$80,000$ income and the proceeds of the $t=0$ investment of $(\$20,000)(1+0.1) = \$22,000$. The amount of money she will have left to consume is $\$80,000 + \$22,000 = \$102,000$.

Mary's consumption now and next year is as follows. **Today:** Consume $\$40,000$. **Next year:** Consume $\$102,000$.

The following graph of Example 3 illustrates how the graph of Example 2 will drastically change when Mary can both borrow and lend at $r=10\%$. Intertemporal consumption choices (1) through (5) of Example 3 are shown on the graph. Choice (1) is just the original “endowment” point “Y” and requires no borrowing or lending to achieve.



The graph above shows the consumption tradeoff possibilities available when Mary can borrow and lend in the financial market at interest rate $r=10\%$. However, any investment made here is just lending at the going market rate of interest, meaning the investment is in *financial* assets at $r=10\%$, as opposed to an investment in a tangible business activity or *real* asset investment.

Example No. 4: Let us extend the example of Mary from Examples 2 and 3 above. Borrowing and lending at $r=10\%$ is available in the financial markets; however, Mary now has a **real investment opportunity or business project**. If Mary decides to accept this opportunity or project, it will cost \$20,000 now ($t=0$) and will offer a *riskless* payoff of \$25,000 next year ($t=1$).

Calculate the rate of return on this one-year project:

$$r_{\text{project}} = (CF_1 - CF_0)/CF_0 = (25,000 - 20,000)/20,000 = 5000/20000 = 0.25 \text{ or } 25\%$$

The 25% return on this real asset investment or business project is superior to the $r=10\%$ return that is available from investing (lending) in financial assets. This project must be taken since it can easily be shown that this project makes Mary wealthier. After all, here money can be borrowed at 10% and invested in a project that earns 25% with no risk.

Now, revisit the “endowment” point (1) from Example 3 above, where Mary has \$60,000 now and will receive \$80,000 next year; however, this real asset project now exists. Assume that she still wants to consume \$60,000 now ($t=0$).

Today (t=0): Consume the existing **\$60,000** today. There is no money left over, so borrow \$20,000 for one year at $r=10\%$ in order to invest in the real project.

Next year (t=1): Mary receives her \$80,000 income and the proceeds of the real investment of \$25,000. She must spend $(\$20,000)(1+0.1) = \$22,000$ in order to repay the loan. The amount of money left to **consume** is $\$80,000 + \$25,000 - \$22,000 = \mathbf{\$83,000}$.

Note the difference from (1) of Example 3. Here, today's (t=0) consumption is the same \$60,000, but next year's consumption is \$3000 greater. Note that \$3000 is left over from the project after the loan is repaid next year. Mary can actually borrow against this \$3000 amount today, and could borrow $3000/(1+0.1) = \underline{\$2727.27}$. This \$2727.27 amount is the project's NPV.

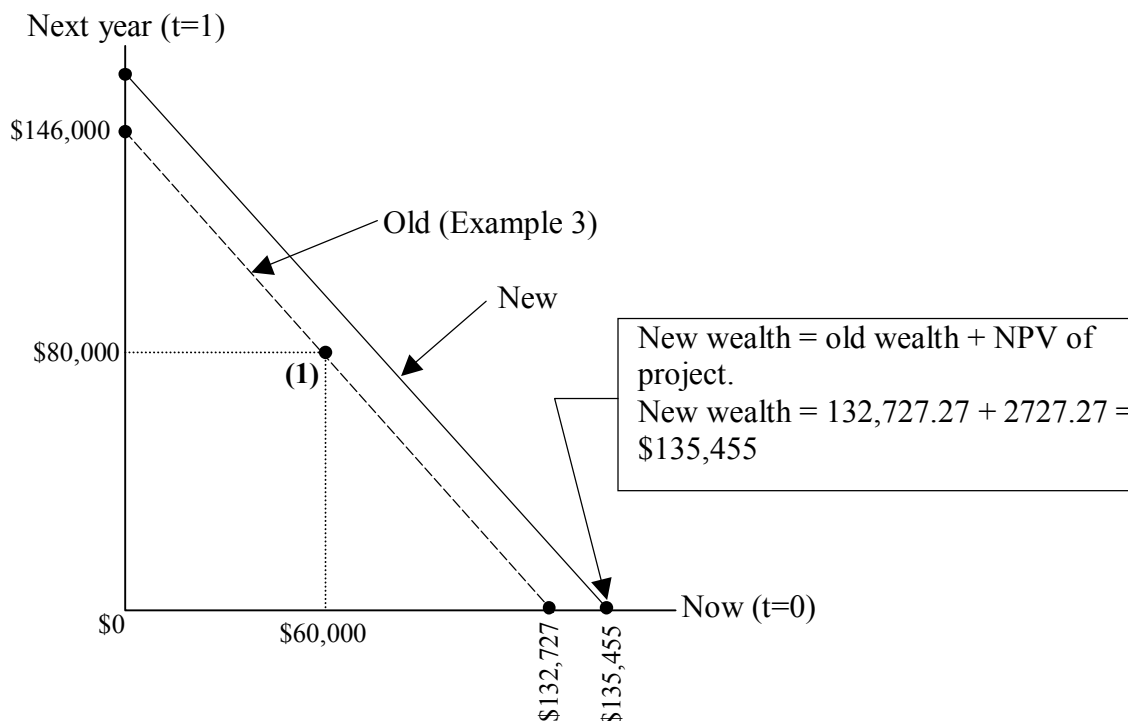
Net Present Value or NPV \equiv Present Value of benefits minus Present Value of costs

Here, the project's NPV = $[25,000/(1+r)] - 20,000 = [25,000/(1+0.1)] - 20,000 = \underline{\$2727.27}$. Here the *cost of capital* for financing this project is $r=10\%$.

This real asset's market value is \$2727.27 more valuable than the resources used to create this asset. The NPV is hence an estimate of the *wealth* that a project creates.

Regardless of Mary's actual consumption preferences, any real investment that has **positive NPV** should always be taken. This leads to the *Separation Theorem* in finance. First, invest to maximize your wealth. Then, borrow or lend to fulfill your consumption needs. Investing and consumption decisions must be separate decisions.

The graph below illustrates the old and new consumption possibilities line for Mary. Point (1) is still the original endowment point "Y" from Examples 2 and 3.



Corporations and Real Investments:

Example No. 5: A corporation consists of a single one-year project. The interest rate for borrowing and lending is $r=10\%$. The corporation has one riskless project; if accepted it will cost \$1,000,000 today. The corporation will be liquidated and produce a certain payoff of \$1,250,000 in exactly one year.

1. The shareholders borrow the \$1,000,000 today at $r=10\%$ and invest in this project.
2. Project pays off \$1,250,000 next year. Shareholders must repay the loan in the amount of $(\$1,000,000)(1+0.1) = \$1,100,000$. Note that $1,250,000 - 1,100,000 = \$150,000$ excess cash remains after the loan is repaid next year.

Today's ($t=0$) Present Value of next year's excess cash amount is $150,000/(1+0.1) = \$136,364$. This is the NPV of this project.

More formally, $NPV = PV(\text{cash outflows}) + PV(\text{cash inflows})$

Here, $NPV = -1,000,000 + 1,250,000/(1+r) = -1,000,000 + 1,250,000/(1+0.1) = \underline{\underline{\$136,364}}$

The shareholders contributed ZERO equity to this firm, so the original *book* value of equity is zero. This single project corporation was originally financed with \$1,000,000 of debt at $r=10\%$. The following table shows today's ($t=0$) balance sheet of this firm using *market* values.

Assets (market value)		Liabilities + Owner's Equity (market value)	
Assets	1,136,364	Debt	1,000,000
		Equity	136,364
Total	1,136,364	Total	1,136,364

The stockholders increase in wealth is equal to the \$136,364 NPV of the project.

Should the consumption preferences of the individual stockholders affect the decision to accept/reject real investments or projects? **Absolutely not!** Here is where we invoke the *Separation Theorem*, where Investment and Consumption decisions are always separate decisions.

Shareholders expect corporations to maximize market value through their investment decisions. The individual shareholders make their consumption decisions separately.