

CHAPTER 15: CAPITAL STRUCTURE (the classic Modigliani-Miller model)

Assigned problems are 6, 12, 13, 14, 15, 17, and 18. The Modigliani-Miller model assumes a *perfect market* — *bankruptcy* costs are not included in this Chapter and thus the debt here is considered free of default risk. Financial distress is covered in Chapter 16.

We first examine an **UNLEVERED** firm (its assets are 100% financed with common stock) that exists in a **NO TAX** world:

	Recession	Normal	Expansion
EBIT	500	1000	1500
-Interest	0	0	0
Net Income	500	1000	1500

In this case of **NO LEVERAGE**, the Recession and Expansion states represent a swing or deviation of 50% from the \$1000 that is expected in a Normal year.

Now, we add leverage to this firm; in which the firm uses some debt to finance its assets (it will have less equity than above). This firm will pay \$500 interest on its debt or bonds.

	Recession	Normal	Expansion
EBIT	500	1000	1500
-Interest	500	500	500
Net Income	0	500	1000

Here, leverage makes the Net Income (and the equity) *more* risky (it also makes the required return of equity, r_E , increase). The Recession and Expansion states represent a -100% and +100% deviation, respectively, from the \$500 expected in a Normal year.

I. CAPITAL STRUCTURE IN A NO TAX WORLD:

\$5000 is invested today in order to produce a firm that will have **perpetual** EBIT of \$1000 per year.¹ The required return on the *assets* of this firm is $r_0 = 10\%$ per year.

We consider TWO methods of financing this firm:

1. **UNLEVERED:** \$5000 of new equity is used to purchase the assets (in any Unlevered case, cost of equity is $r_E = r_0$).

¹ For simplicity, here we model all cash flows as perpetuities, where $PV_0 = CF/r$. The market value of any perpetual bond is equal to $PV_0 = r_B B / r_D$.

2. **LEVERED:** Assets are purchased with \$4000 of new equity and \$1000 of new debt or bonds (perpetual debt, with a face value of $B=\$1000$ and coupon rate on this debt of $r_B = 5\%$). Interest cost will be $r_B B = [0.05][1000] = \$50$ per year in perpetuity. Let $r_D=5\%$.

Note: r_D is the current *market* required rate of return on the bonds. r_D can certainly differ from r_B since coupon rates never change, while the market required rate changes over time.

The **perpetual** cash flows are the following for the **Unlevered** and **Levered** scenarios:

	<u>UNLEVERED</u>	<u>LEVERED</u>
EBIT	1000	1000
-Interest	0	50
Earnings before taxes	1000	950
-Tax	0	0
Net Income	1000	950

Unlevered firm: pays out **EBIT = \$1000** to equity each year.

Levered firm: pays out **[EBIT - $r_B B$] = \$950** to equity and **$r_B B = \50** to bonds; for a total of **\$1000** each year paid out by the firm.

The *total* annual amount of cash paid out by the firm to its security holders in each scenario is **\$1000**, and is thus **unchanged** or not affected by leverage in a **NO TAX** world.

Value of an all equity or UNLEVERED firm in a NO TAX world:

The required return to equity r_E is equal to r_0 only for an *unlevered* firm.

$$V_U = [\text{CF to equity}]/r_0 = \text{EBIT}/r_0 = 1000/0.10 = \mathbf{\$10,000}$$
, total value of equity

This firm is 100% equity financed at $r_E = 10\%$. Its **Weighted Average Cost of Capital** is:

$$\text{WACC} = [D/(D+E)][1 - T_c][r_D] + [E/(D+E)][r_E]$$

$$\text{WACC} = [0/(0 + 10000)][1 - 0][r_D] + [10000/(0 + 10000)][0.10] = 0.10 \text{ or } \mathbf{10\%}$$

Value of a LEVERED firm in a NO TAX world: Let $r_D = r_B$ on the firm's debt

$$V_L = E + D = [\text{CF to equity}]/r_E + [\text{CF to bonds}]/r_D = 950/r_E + 50/r_D$$

Equity plus debt, $E + D$, is always the total market value of the levered firm or V_L . Remember that in Finance, E and D are always the current *market* values of equity and debt.

We know that $r_0 = 10\%$ and $r_B = r_D = 5\%$, but we don't yet know what r_E is equal to (it will certainly be greater than r_0 in any Leveraged firm).

Total cash flow paid out by the firm is **[EBIT - $r_B B$] + $r_B B = \text{EBIT}$**

In a **NO TAX** world, Leverage has no effect on the total cash flow that the firm is able to pay out to both equity and debt. The **total** cash flow paid out is equal to EBIT and this EBIT = \$1000 cash flow has a required return of $r_0 = 10\%$. Why is this? The EBIT produced by the Assets of this firm is *independent* of how the firm is financed. In the **Unlevered** case, you can own all of the equity and receive \$1000 per year. In the **Levered** case you can own all of the equity and bonds of this firm and still receive a total of \$1000 per year.

In a **NO TAX** world: $V_L = \text{EBIT}/r_0 = 1000/0.10 = \$10,000$, the same value as V_U

Modigliani-Miller Proposition #1: In a **NO TAX** world, $V_L = V_U$. Leverage does not affect the firm's total value.

Modigliani-Miller Proposition #2 for the NO TAX world:

1. $r_E = r_0 + [r_0 - r_D][D/E]$. As the debt to equity ratio increases (leverage increases), the required return to equity increases, i.e., increasing leverage makes the firm's equity more risky and therefore r_E must increase. The r_0 is *independent* of leverage, but r_E is not!
2. $\text{WACC} = r_0 = [D/(D+E)][r_D] + [E/(D+E)][r_E]$. In this case, the WACC is both *constant* and *fixed* at r_0 .

Here, $r_0 = 10\%$ and $r_B = r_D = 5\%$. The values of the equity (E) and debt (D), and r_E are easily calculated. First, consider the perpetual debt (face value B = \$1000):

$$D = [\text{CF to debt}]/r_D = r_B B / r_D = [(0.05)(1000)]/0.05 = 50/0.05 = \mathbf{\$1000}$$

$$E = V_L - D = 10,000 - 1000 = 9000, \text{ the levered equity is worth } \mathbf{\$9000}$$

$$r_E = r_0 + [r_0 - r_D][D/E] = 0.10 + [0.10 - 0.05][1000/9000] = 0.105555 \text{ or } \mathbf{10.5555\%}$$

$$\text{So, } E = [\text{CF to equity}]/r_E = [\text{EBIT} - r_B B]/r_E = 950/0.105555 = \mathbf{\$9000} \text{ (already known!)}$$

Another important relation is: $V_L = [\text{unlevered cash flow to equity}]/\text{WACC} = \text{EBIT}/\text{WACC} = 1000/0.10 = \$10,000$. In a no tax world, $\text{WACC} = r_0$.

Note: This firm costs \$5000 to create, but it has a total value of \$10,000. Creating this firm has an **NPV** = $-5000 + 10000 = \mathbf{\$5000}$.

II. CAPITAL STRUCTURE IN A WORLD WITH TAXES:

We use the same firm as before, except now we allow for a corporate income tax ($T_C = 34\%$). Taxes are assessed on the Earnings before taxes. Cash paid to bonds and equity is in bold.

	<u>UNLEVERED</u>	<u>LEVERED</u>
EBIT	1000	1000
-Interest	0	50
Earnings before taxes	1000	950
-Tax	340	323
Net Income	660	627

Here, as in all previous examples of this firm, the required return on the firm's assets is $r_0 = 10\%$ (r_0 is not dependent on how the firm is financed or if it is taxed).

What changes with Leverage when you have taxes?

1. The **Unlevered** (all equity) firm pays out $EBIT[1 - T_C] = \mathbf{\$660}$ each year to equity. This cash flow must be discounted at $r_0 = 10\%$ the *unlevered* or *all equity* required rate of return.
2. The **Levered** firm pays out $\mathbf{\$627}$ to equity and $\mathbf{\$50}$ of interest to the bonds, for a total of $627 + 50 = \mathbf{\$677}$ each year to all security holders. This is $\mathbf{\$17}$ more than the Unlevered firm can pay out. Where does the \$17 come from? It comes from paying $r_B B T_C = \$17$ less in taxes each year, since the cost of interest payments is tax deductible. The \$17 is called the *tax shield* of debt.

Values of (1) Unlevered (easy) and (2) Levered firms (complicated):

(1) Unlevered firm: $V_U = E = EBIT[1 - T_C]/r_0 = 660/0.10 = \mathbf{\$6600}$, where $r_E = r_0$.

(2) Levered firm: as always, $V_L = E + D \rightarrow V_L = [EBIT - r_B B][1 - T_C]/r_E + r_B B/r_D$

The equity receives: $[EBIT - r_B B][1 - T_C] = [1000 - 50][0.66] = \mathbf{\$627}$ per year. This amount is discounted at the levered equity required return r_E , which *we don't yet know*.

The bond receives: $r_B B = [0.05][1000] = \mathbf{\$50}$ of interest paid per year. This amount is discounted at r_D , the market required rate of return on the debt.

★ The *levered* firm pays out a *total* amount to equity and bonds of: $[EBIT - r_B B][1 - T_C] + r_B B$.

This relation can be easily algebraically rearranged to obtain: $[EBIT][1 - T_C] + r_B B T_C$, where:

★ The first term, $[EBIT][1 - T_C] = 1000[0.66] = \mathbf{\$660}$, and is identical to the cash flow to the **Unlevered** equity. This cash flow must be discounted at the required return of *unlevered* equity or assets $r_0 = 10\%$ in this example.

★ The second term, $r_B B T_C = (0.05)(1000)(0.34) = \mathbf{\$17}$, is the *tax shield* of debt, or annual tax savings due to the tax deductibility of interest on the debt. This cash flow must be discounted at the required return on the debt $r_D = 5\%$.

Thus the *total* cash flow paid out to investors by this *Levered* firm is $\$660 + \$17 = \$677$.




Using the first term and second term from above and the appropriate discount rates for each term, the value of the leveraged firm can be represented as follows:

$$V_L = V_U + \text{PV of tax shield}$$

$$V_L = \text{EBIT}[1 - T_C]/r_0 + r_B B T_C / r_D$$

$$V_L = 1000[0.66]/0.10 + (0.05)(1000)(0.34)/0.05 = \underline{\underline{6600}} + \underline{\underline{340}} = \underline{\underline{6940}}$$

The value of the Unlevered firm is $\$6600$ and the Present Value of the tax shield of debt is $\$340$. When you add leverage to any firm, its value will increase only by the amount of the tax shield of debt. In this example, the Levered firm is worth \$340 more than the Unlevered version.

If $r_B = r_D$, then $D = B$. Thus in this particular example we have: $V_L = V_U + T_C B$. 

Next, we find the value of the equity of this levered firm:

$$V_L = E + D, \text{ where } D = r_B B / r_D = (0.05)(1000)/0.05 = 50/0.05 = \underline{\underline{1000}}$$

$$\text{Since } V_L = E + D, \text{ then } E = V_L - D = 6940 - 1000 = \underline{\underline{\$5940}}$$

Next, we look at the other equation of determining the value of a Levered firm:

$$V_L = E + D = [\text{EBIT} - r_B B][1 - T_C]/r_E + r_B B / r_D = 627/r_E + 50/r_D$$

V_L obviously is \$6940 for this example, but we can't use this equation to find V_L since we still don't know r_E . However, we can certainly use 6940 for V_L and 1000 for D and solve for r_E .

For the world *with* taxes:

1. $r_E = r_0 + [r_0 - r_D][1 - T_C][D/E] \rightarrow$ As the debt to equity ratio increases (leverage increases), the required return to equity increases, i.e., increasing leverage makes the firm's equity more risky and therefore r_E must increase.

2. $\text{WACC} = [D/(D+E)][1 - T_C][r_D] + [E/(D+E)][r_E] \rightarrow$ WACC is *not* constant.

$$r_E = 0.10 + [0.10 - 0.05][1 - 0.34][1000/5940] = 0.105555.$$

Plug this r_E into $E = [\text{EBIT} - r_B B][1 - T_C]/r_E$ and you get $E = \underline{\underline{\$5940}}$ (which already know)

$$\text{WACC} = [1000/(1000+5940)][1 - 0.34][0.05] + [5940/(1000+5940)][0.105555] = 0.0951 \text{ or } \underline{\underline{9.51\%}}.$$

Also note that with perpetual cash flows:

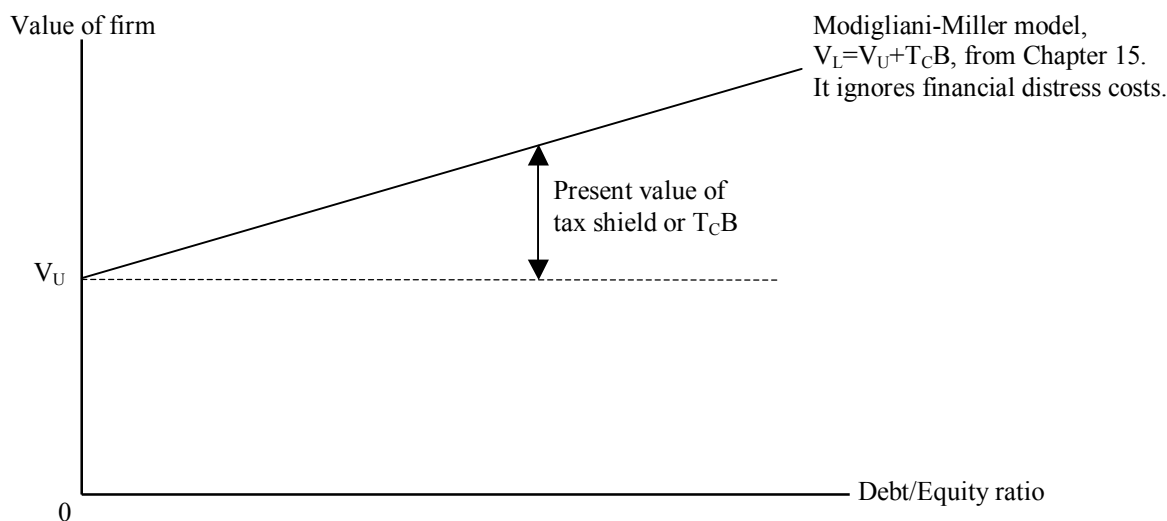
$$V_L = [\text{Unlevered cash flow}]/\text{WACC} = \text{EBIT}[1 - T_C]/\text{WACC} = 660/0.0951 = \$6940$$



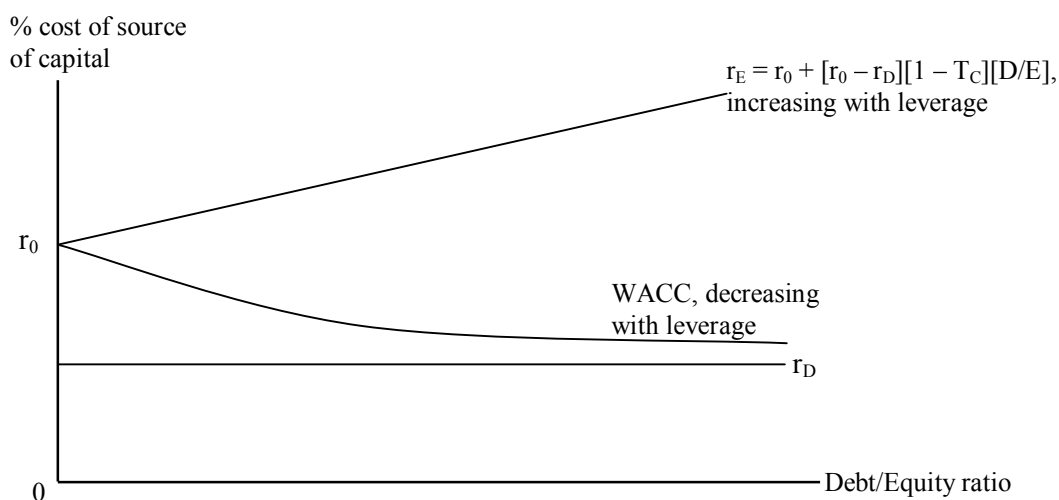
Note that: $\text{WACC} = \text{EBIT}[1 - T_C]/V_L$. This is a short cut to finding WACC.

Modigliani and Miller for leverage in a world with taxes:

Proposition #1: $V_L = V_U + \text{PV of Tax Shield}$. The value of a levered firm equals the Unlevered value plus the PV of the tax shield of debt (the PV of the perpetuity of annual tax savings). If you have perpetual debt, and if $r_B = r_D$, then $V_L = V_U + T_C B$.



Proposition #2: WACC *decreases* as a greater proportion of the firm is financed with debt, i.e., as the ratio D/E increases. For a *Levered* firm, WACC is less than r_0 . This model ignores the costs of bankruptcy and financial distress since the model suggests that WACC is *minimized* at being 100% debt financed. This result is obviously too simplistic and this topic is further addressed in Chapter 16, where the limits to debt are addressed.



III. EXAMPLE OF A LEVERAGED RECAPITALIZATION², USING THE EXAMPLES FROM THIS HANDOUT:

The firm has perpetual EBIT = \$1000 per year

Assets have a required return of $r_0 = 10\%$

Tax rate is $T_C = 34\%$

Perpetual debt can be issued at $r_D = 5\%$. The coupon rate will be $r_B = r_D = 5\%$, and thus $D=B$.

This firm is currently **all equity** financed or **Unlevered**. The firm's equity consists of 1000 shares of outstanding common stock.

Currently the value of this firm is $E = V_U = \text{EBIT}[1 - T_C]/r_0 = 660/0.10 = \mathbf{\$6600}$

Stock price = $E/[\text{number of shares}] = 6600/1000 = \mathbf{\$6.60 \text{ per share}}$

Now the firm announces that it will become Levered. It will issue $B=\$1000$ par value of debt and use the proceeds of the debt issue to buy back \$1000 of stock.

At the announcement, the value of this firm should now increase by the PV of the tax shield (an Efficient Market should immediately incorporate the imminent debt and tax shield into the firm's value and stock price). This firm will be paying \$17 less in taxes each year ($17/0.05 = \$340$).

PV of tax shield = $r_B T_C B / r_D = T_C B = [0.34][1000] = \340

The debt has not yet been issued, however, the value of the firm (and it is still all equity or Unlevered) should go from \$6600 to \$6940 at the announcement.

New stock price = $6940/(1000 \text{ shares}) = \6.94 per share . The stock will rise in price from \$6.60 to \$6.94 at the announcement.

The firm will buy back stock for \$6.94 per share. With \$1000 you can repurchase $1000/6.94 = 144.0922$ shares. Now, $1000 - 144.0922 = 855.90778$ shares remain.

The new Levered firm will consist of 855.90778 shares worth \$6.94 each and one bond or debt issue worth \$1000:

$E = (855.90778)(6.94) = \5940 and $D = \$1000$

$V_L = E + D = 5940 + 1000 = \6940

All of the shareholders benefit from this announcement, whether they keep or sell their shares. Those that keep their shares now have shares worth \$6.94. Those that sell shares will sell them for \$6.94.

² In a leveraged recapitalization, debt is issued and the proceeds of the issue are used to buy back common stock. A leveraged recapitalization is a capital structure or financial change, as the assets of the firm are not affected.